MATH-480 Tensor Calculus

Credit Hours: 3-0

Prerequisite: None

Course Objectives: The purpose of this course is to introduce the basic definitions and techniques arising in tensor calculus, differential geometry and continuum mechanics. In particular, this course is aimed to (i) develop a physical understanding of the mathematical concepts associated with tensor calculus and (ii) develop the basic equations of tensor calculus, differential geometry and continuum mechanics which arise in applications.

Core Contents: Index notation, Transformation equations, Contravariant components, Special tensors: Metric tensor, Derivative of a tensor, Differential geometry and relativity, Tensor notation for scalar and vector quantities, Basic equations of continuum mechanics.

Detailed Course Contents: Index notation: Symmetric and skew-symmetric systems, Summation convention, Addition, multiplication and contraction, The e-permutation symbol and Kronecker delta, The e-delta Identity, Generalized Kronecker delta, Additional applications ofthe indicial notation.

Transformation equations, Calculation of derivatives, Vector identities in Cartesian coordinates, Indicial Form of Integral Theorems, Determinants, Cofactors

Tensor concepts and transformations: Reciprocal Basis, Coordinate Transformations, Scalars, Vectors and Tensors, Cartesian Coordinates, Scalar functions and invariance, Vector Transformation, Contravariant components, Covariant components, Higher order tensors, Dyads and polyads, Operations using tensors(Addition and Subtraction, Outer Product, Contraction, Inner product, Quotient law)

Special tensors: Metric tensor, Conjugate metric tensor, Associated tensors, Riemann space, Epsilon Permutation Symbol, Cartesian

Tensors, Physical Components, Physical Components For Orthogonal Coordinates, Higher order tensors, Physical components in general, Tensors and multilinear forms, Dual Tensors, Derivative of a tensor: Christoffel Symbols, Covariant differentiation, Covariant derivative of contravariant tensor, Rules for covariant differentiation, Riemann Christoffel tensor, Physical Interpretation of Covariant Di_erentiation, Ricci's theorem, Intrinsic or absolute differentiation, Parallel vector fields,

Differential geometry and relativity: Surfaces and Curvature, Normal Curvature, The equations of Gauss. Weingarten and Codazzi, Geodesic Curvature, Tensor derivatives, GeodesicCoordinates, Riemann Christoffel tensor. Surface Curvature.

Tensor notation for scalar and vector quantities: Gradient, Divergence, Laplacian, Eigenvalues and Eigenvectors of Symmetric Tensors. Dynamics: Particle Movement, Frenet-Serret formulas, Work and Potential Energy, Conservative systems. Lagrange's equations of motion, Euler- Lagrange equations of motion, Action Integral, Dynamics of Rigid Body Motion, Relative motion and angular velocity, Euler's equations of motion.

Basic equations of continuum mechanics: Introduction to elasticity, Normal and shearing stresses, The stress tensor, Cauchy stress law. Conservation of linear momentum, Conservation of angular momentum, Strain in Two Dimensions. Transformation of an arbitrary element, Cartesian tensor derivation of strain. Lagrangian and Eulerian Systems, General tensor derivation

of strain, Compressible and incompressible material, Conservation of mass.

Course Outcomes: The students are expected to:

- Develop a physical understanding of the mathematical concepts associated with tensorcalculus and.
- Develop the basic equations of tensor calculus, differential geometry and continuummechanics which arise in applications.

Text Books: J. H. Heinbockel, Introduction to Tensor Calculus and Continuum Mechanics,Trafford Publishing (2001)

Reference Books:

1. Mikhail Itskov, Tensor algebra and tensor analysis for engineers with applications tocontinuum mechanics, Springer Berlin Heidelberg (2009).
2. Theodore Frankel, The Geometry of Physics, An Introduction (3rd Ed.), Cambridge (1997).

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1	Index notation: Symmetric and skew-symmetric systems, Summation convention, Addition, multiplication and contraction, The e-permutation symbol and Kronecker delta, The e-delta Identity, Generalized Kronecker delta, Additional applications of the indicial notation.
2	1.1 (cont.)	Transformation equations, Calculation of derivatives, Vector identities in Cartesian coordinates, Indicial Form of Integral Theorems, Determinants, Cofactors
3	1.2	Tensor concepts and transformations: Reciprocal Basis, Coordinate Transformations, Scalars, Vectors and Tensors, Cartesian Coordinates, Scalar functions and invariance, Vector transformation,
4	1.2 (cont.)	Contravariant components, Covariant components, Higher order tensors, Dyads and polyads, Operations using tensors (Addition and Subtraction, Outer Product, Contraction, Inner product, Quotient law).
5	1.3	Special tensors: Metric tensor, Conjugate metric tensor, Associated tensors, Riemann space, Epsilon Permutation Symbol, Cartesian.

6	1.3 cont.	Tensors, Physical Components, Physical Components For Orthogonal Coordinates, Higher order tensors, Physical components in general, Tensors and multilinear forms, Dual Tensors.
7	1.4	Derivative of a tensor: Christoffel Symbols, Covariant differentiation, Covariant derivative of contravariant tensor, Rules for covariant differentiation, Riemann Christoffel tensor, Physical Interpretation of Covariant Differentiation, Ricci's theorem, Intrinsic or absolute differentiation, Parallel vector fields.
8	1.5	Differential geometry and relativity: Surfaces and Curvature, Normal Curvature, The equations of Gauss.
9	**Mid Semester Exam**	
10	1.5 cont.	Weingarten and Codazzi, Geodesic Curvature, Tensor derivatives, Geodesic Coordinates, Riemann Christoffel tensor.
11	1.5 Cont.2.1	Surface Curvature, Tensor notation for scalar and vector quantities: Gradient, Divergence, Laplacian, Eigenvalues and Eigenvectors of Symmetric Tensors.
12	2.2	Dynamics: Particle Movement, Frenet-Serret formulas, Work and Potential Energy, Conservative systems.
13	2.3	Basic equations of continuum mechanics: Introduction to elasticity, Normal and shearing stresses, The stress tensor, Cauchy stress law.
14	2.3 cont.	Conservation of linear momentum, Conservation of angular momentum, Strain in Two Dimensions.
15	2.3 cont.	Transformation of an arbitrary element, Cartesian tensor derivation of strain.
16	2.3 cont.	Lagrangian and Eulerian Systems, General tensor derivation of strain, Compressible and incompressible material, Conservation of mass.
17		Revision
18	**End Semester Exam**	